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CALCULATION OF LINEAR ELASTIC FOR SHAFTS ASYMMETRICALLY AND SYMMETRICALLY MECHANISM

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Abstract: Based on work [3], the work below presents the algorithm for calculating the cross shaft mechanism, as both the symmetric and asymmetric mechanism without taking the Judge must also consider technical violations. Preparatory work is the calculation approach of linear elastic shafts with technical violations.

1. INTRODUCTION

This paper develops mathematical algorithm for calculating the reaction forces that arise in building components of the mechanism cross shaft using linear elastic calculation, a method that is based on relative displacements and coordinates plückeriene.

2. THE MECHANISM ASYMMETRICALLY

2.1 THE MATHEMATICAL MODEL

Asymmetric mechanism is considered in figure 1, in a given position (θ_1, θ_2-angles known) general reference system fixed load $OX_0Y_0Z_0$ and the $\{F\}$ last element.

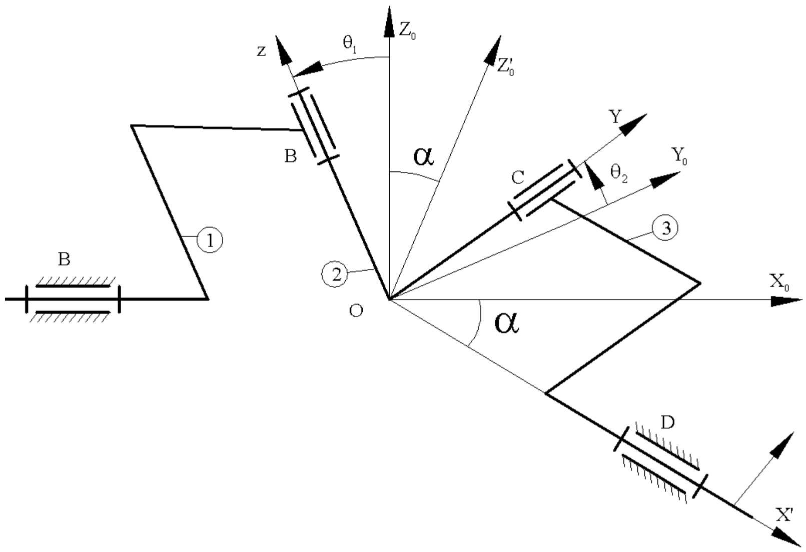

Figure 1. Asymmetric mechanism

By blocking the joint A, noting the efforts of A and $\{F_A\}$ with $\{F_B\},\{F_C\},\{F_D\}$ reactions of the couplings B,C,D of the balance equations, taking into account the principle of action and reaction, to obtain equations

$$\{F_A\}-\{F_B\}=\{0\},\{F_B\}-\{F_C\}=\{0\},\{F_C\}-\{F_D\}+\{F\}=\{0\} \tag{2.1}$$

Rotational kinematics couplings of B,C,D are defined [2] the column matrices of coordinates plückeriene.

$$\{U_B\} = [0 \quad -\sin\theta_1 \quad \cos\theta_1 \quad 0 \quad 0 \quad 0]^T \ ;$$
$$\{U_C\} = [\sin\theta_2 \sin\alpha \quad \cos\theta_2 \quad \sin\theta_2 \cos\alpha \quad 0 \quad 0 \quad 0]^T \ ; \tag{2.2}$$
$$\{U_D\} = [\cos\alpha \quad -\sin\alpha \quad 0 \quad 0 \quad 0 \quad 0]^T \ ;$$

where the angle θ_2, resulting in kinematic conditions [1] and verifies the equation

$$\mathrm{tg}\theta_2 = \frac{\mathrm{tg}\theta_1}{\cos\alpha} \tag{2.3}$$

conditions that the kinematic coupling B, C, D to be written as rotation

$$\{\tilde{U}\} = [\{\tilde{U}_B\} \quad \{\tilde{U}_C\} \quad \{\tilde{U}_D\}]^T \tag{2.4}$$

equality is deduced

$$\{\tilde{U}\}\{F_A\} = [0 \quad 0 \quad -\{\tilde{U}_D\}\{F\}]^T \tag{2.5}$$

Next, using the relation (2.3) from [3], to write the equalities

$$\{F_A\} = -[K_{AB}]\{\Delta_{B_1}\}; \ \{F_B\} = -[K_{BC}]\{\{\Delta_{B_2}\} - \{\Delta_{C_2}\}\}; \ \{F_C\} = -[K_{CD}]\{\{\Delta_{B_3}\} - \{\Delta_{D_3}\}\} \tag{2.6}$$

plus the relationship between displacements

$$\{\Delta_{B_2}\} = \{\Delta_{B_1}\} + \xi_B\{U_B\}; \ \{\Delta_{C_3}\} = \{\Delta_{C_2}\} + \xi_C\{U_C\}; \ \{0\} = \{\Delta_{D_3}\} + \xi_D\{U_D\} \tag{2.7}$$

where the notation has been noted 11 spins elastic kinematic couplings.

From equations (2.5), (2.6), (2.7) using the notation

$$\{\xi\} = [\xi_B \quad \xi_C \quad \xi_D]; \ [U] = [\{U_B\} \quad \{U_C\} \quad \{U_D\}]; \tag{2.8}$$

$$[H_{AD}] = [H_{AB}] + [H_{BC}] + [H_{CD}]; \ [K_{AD}] = [H_{AD}]^{-1} \tag{2.9}$$

order to obtain the equalities

$$\{\xi\} = [[\tilde{U}][K_{AD}][U]]^{-1}[0 \quad 0 \quad -[\tilde{U}_D]\{F\}]^{-1} \ ; \ \{F_A\} = [K_{AD}][U]\{\xi\} \tag{2.10}$$

and equations (2.5) from [3], is deducted then the reactions $\{F_B\}$, $\{F_C\}$, $\{F_D\}$.

3. 2. STIFFNESS MATRICES. FLEXIBILITY MATRICES

Reference systems is considered (fig. 1)

- . OX_0Y^*Z - obtained by rotating around the axis OX_0 of $OX_0Y_0Z_0$ system with the angle θ_1;
-. OX^*YZ with axis OX^* perpendicular to the axis OY, OZ;
-. $OX'YZ_0^*$ - obtained by rotating around the axis OX' of $OX'Y_0Z_0'$ $(OZ_0' \perp OX')$ system by angle θ_2.

Flexible arrays $[H_{AB}^*]$, $[H_{BC}^*]$, $[H_{CD}^*]$ respectively in relation to these reference systems are constant matrices and matrices of flexibility to fixed reference system $OX_0Y_0Z_0$ is generally expressed by relations

$$[H_{AB}] = [T_{AB}][H_{AB}^*][T_{AB}]^{-1} \ ; \ [H_{BC}] = [T_{BC}][H_{BC}^*][T_{BC}]^{-1} \ ; \ [H_{CD}] = [T_{CD}][H_{CD}^*][T_{CD}]^{-1} \tag{2.11}$$

where

$$[T_{AB}] = \begin{bmatrix} [R_{AB}] & [0] \\ [0] & [R_{AB}] \end{bmatrix}^{-1} \ ; \ [T_{BC}] = \begin{bmatrix} [R_{BC}] & [0] \\ [0] & [R_{BC}] \end{bmatrix}; \ [T_{CD}] = \begin{bmatrix} [R_{CD}] & [0] \\ [0] & [R_{CD}] \end{bmatrix} \tag{2.12}$$

$$[R_{AB}]=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta_1 & -\sin\theta_1\\ 0 & \sin\theta_1 & \cos\theta_1\end{bmatrix};[R_{CD}]=\begin{bmatrix}\cos\alpha & \sin\alpha\sin\theta_2 & \sin\alpha\cos\theta_2\\ 0 & \cos\theta_2 & -\sin\theta_2\\ -\sin\alpha & \cos\alpha\sin\theta_2 & \cos\alpha\cos\theta_2\end{bmatrix}$$

$$[R_{BC}]=\begin{bmatrix}\cos\theta_1\cos\theta_2+\sin\theta_1\sin\theta_2\cos\alpha & \sin\alpha\sin\theta_2 & 0\\ -\cos\theta_1\sin\theta_2\sin\alpha & \cos\theta_2 & -\sin\theta_1\\ -\sin\theta_1\sin\theta_2\sin\alpha & \cos\alpha\sin\theta_2 & \cos\theta_1\end{bmatrix}; \tag{2.12}$$

Still be considered in order sections (figure. 1)

- $AA', A'A'', A''B$, denoted with indices $1, 2, 3$, with the general reference OX_0Y^*Z and local systems (fig. 2. a, b, c) so that $Ax_1 \| OX_0$; $Az_1 \| OZ$; $A'x_2 \| OZ$; $A'z_2 \| OZ_0$; $A''x_3 \| OX_0$; $A''z_3 \| OZ$;

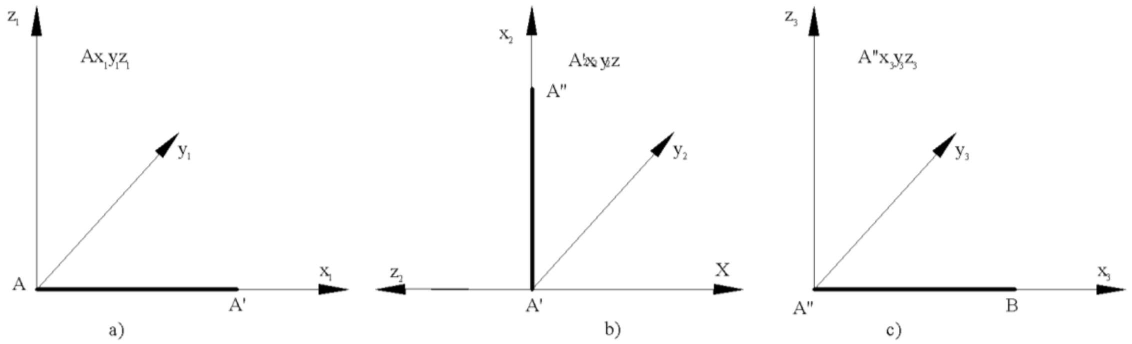

Figure 2. Sections $AA', A'A'', A''B$

- OB, OC, denoted with indices $4, 5$, with the general reference OX^*YZ and local systems (fig. 3. a, b), $Ox_4y_4z_4$, $Ox_5y_5z_5$ so that Ox_4 coincides with OY and Oy_4 coincides with OY; Ox_5 coincides with OY and Oy_5 coincides with OZ;

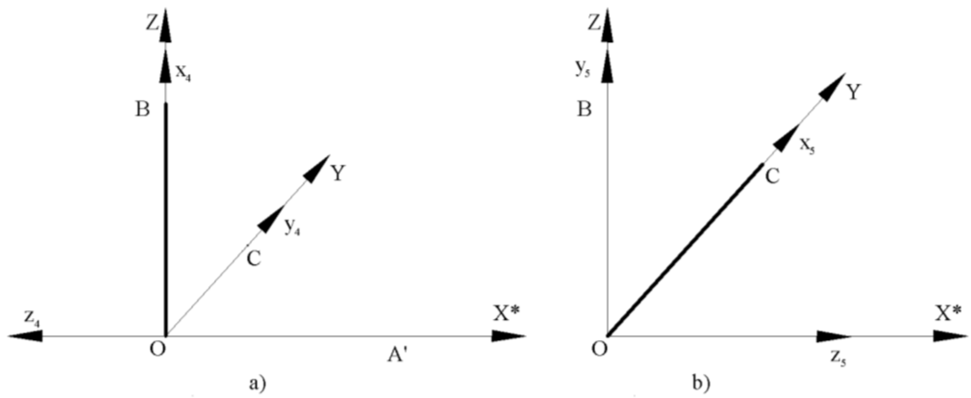

Figure 3. Sections OB, OC

- CD'', $D''D'$, $D'D$, denoted by index $6, 7, 8$, with the general reference $OX'YZ_0^*$ and local systems (fig. 4. a, b, c) so that $Cx_6 \| OX'$, $Cz_6 \| OY$; $D'x_7 \| OZ_0^*$; $D'y_7 \| OY$; $D'x_8 \| OX'$; $D'z_8 \| OY$.

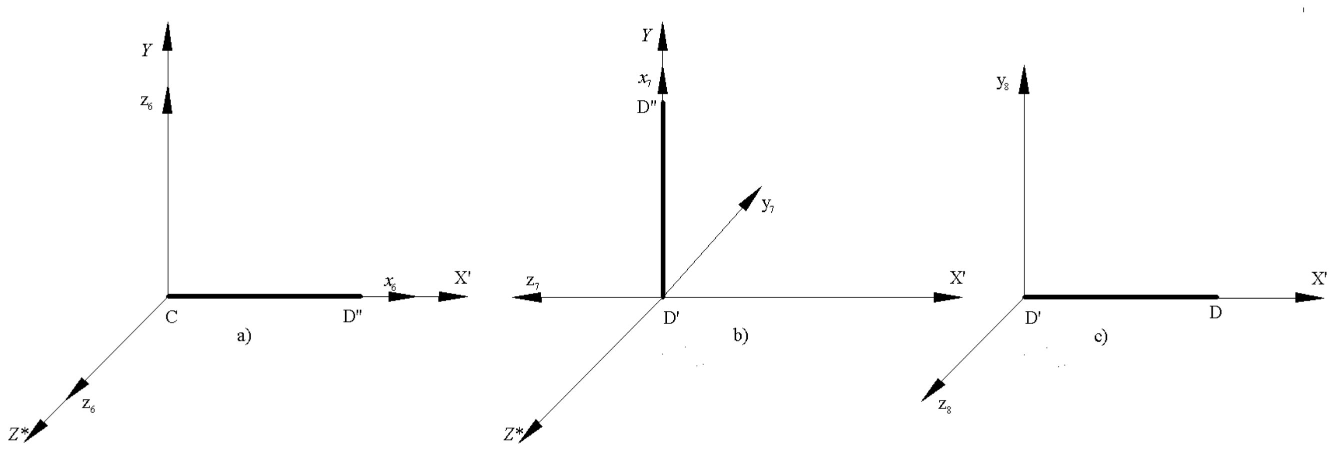

Figure 4. Sections CD'', $D''D'$, $D'D$

Noting with l_i, A_i, I_{ix}, I_{iy}, I_{iz}, E_i, G_i, $i=1,2,3,.....,8$, geometrical and mechanical characteristics of a section, then local flexibility matrices $[h_i]$ is determined by the second relation (2.5) from [3], and constant flexibility matrices $[H_i^*]$ reported that OX_0Y^*Z, OX^*YZ, $OX'YZ_0^*$ systems, are given by relations

$$[H_i^*]=[T_i^*][h_i][T_i^*]^{-1},\ i=1,2,3,.....,8 \tag{2.13}$$

where

$$[T_i^*]=\begin{bmatrix}[R_i] & [0]\\ [G_i][R_i] & [R_i]\end{bmatrix},\ [T_i^*]^{-1}=\begin{bmatrix}[R_i]^T & [0]\\ [G_i]^T[R_i]^T & [R_i]^T\end{bmatrix};$$

$$[R_1]=[R_3]=[I];$$

$$[R_2]=\begin{bmatrix}0&0&-1\\0&1&0\\1&0&0\end{bmatrix};\ [R_4]=\begin{bmatrix}0&0&-1\\0&1&0\\1&0&0\end{bmatrix};\ [R_5]=\begin{bmatrix}0&0&1\\1&0&0\\0&1&0\end{bmatrix} \tag{2.14}$$

$$[R_6]=[R_8]=[I];\ [R_7]=\begin{bmatrix}0&0&-1\\1&0&0\\0&-1&0\end{bmatrix} \tag{2.15}$$

$$[G_i]=\begin{bmatrix}0&-Z_i&Y_i\\Z_i&0&-X_i\\-Y_i&X_i&0\end{bmatrix},\ i=1,2,3,.....,8 \tag{2.16}$$

$$X_1=-(l_1+l_3);\ Y_1=Z_1=0;\ X_2=-l_3;\ Y_2=Z_2=0;\ X_3=-l_3;\ Y_3=0;\ Z_3=l_2;$$
$$X_4=X_5=Y_4=Y_5=Z_4=Z_5=0;\ X_6=0;\ Y_6=l_7;\ Z_6=0;\ X_7=X_8=l_6; \tag{2.17}$$
$$Y_7=Y_8=Z_7=Z_8=0.$$

Flexibility matrices constant $[H_{AB}^*]$, $[H_{BC}^*]$, $[H_{CD}^*]$ is defined by relations

$$[H_{AB}^*]=[H_1^*]+[H_2^*]+[H_3^*];\ [H_{BC}^*]=[H_4^*]+[H_5^*];\ [H_{CD}^*]=[H_6^*]+[H_7^*]+[H_8^*] \tag{2.18}$$

and then the relations (2.9) - (2.12) is calculated for given values of grade θ_1 degree angle, variable arrays $[H_{AB}]$, $[H_{BC}]$, $[H_{CD}]$,, necessary to determine the matrix $[\xi]$, $[F_A]$, the relations (2.9). (2.10).

2.3. THE CALCULATION ALGORITHM

- are known geometrical and mechanical characteristics of l_i, A_i, I_{ix}, I_{iy}, I_{iz}, E_i, G_i, $i = 1,2,3,.....,8$, is determined by relations (2.5) from [3], (2.13) - (2.18) constant matrices $[H^*_{AB}]$, $[H^*_{BC}]$, $[H^*_{CD}]$;
- determine the position arrays $[T_{AB}]$, $[T_{BC}]$, $[T_{CD}]$ and $[H_{AB}]$, $[H_{BC}]$, $[H_{CD}]$ flexibility matrix for each value of angle $[\theta_1]$, the relations (2.7) from [3], (2.11), (2.12), (2.13) ;
- determine the matrices $\{U_B\}$, $\{U_C\}$, $\{U_D\}$, $[\tilde{U}_B]$, $[\tilde{U}_C]$, $[\tilde{U}_D]$, $[U]$, $[\tilde{U}]$ with relations (2.2), (2.4), (2.6), (2.8) ;
- determine the matrix displacements of kinematic couplings with (2.10) and $\{F_A\}$ with reaction matrix equation (2.9) ;
- determine the reactions of $\{F_B\}$, $\{F_C\}$, $\{F_D\}$, and relations (2.1).

3. CROSS SHAFT. SYMMETRICALLY MECHANISM

For this mechanism to consider the representation and notation of fig. 5, the kinematic coupling of A blocked

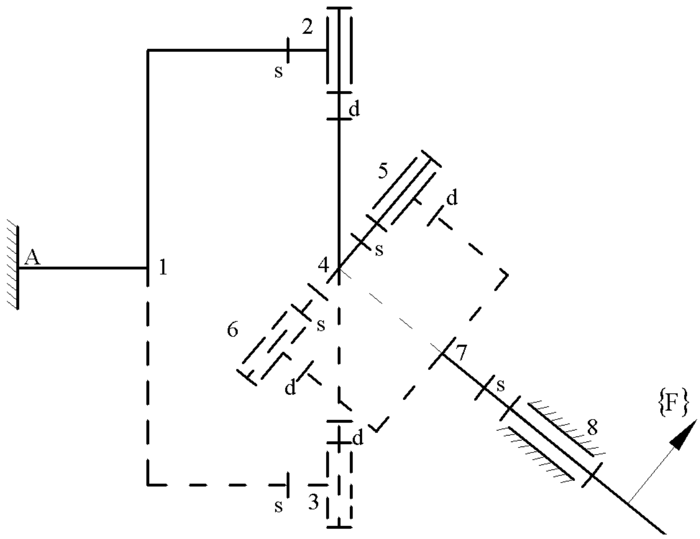

Figure 5. Symmetrically mechanism

Noting with $\{\Delta_1\}$, $\{\Delta_4\}$, $\{\Delta_7\}$, displacements 1, 4, 7, knots,, with, $\{\Delta^s_2\}$, $\{\Delta^d_2\}$, $\{\Delta^s_3\}$, $\{\Delta^d_3\}$, $\{\Delta^s_5\}$, $\{\Delta^d_5\}$, $\{\Delta^s_6\}$, $\{\Delta^d_6\}$, $\{\Delta^s_8\}$ movements of left and right sections of rotational kinematics couplings 2, 3, 5, 6, 7 with ξ_2, ξ_3, ξ_5, ξ_6, ξ_7 shifts in the same kinematic couplings relations can be written

$$\{\Delta^d_2\} = \{\Delta^s_2\} + \xi_2\{U_2\} \ ; \ \{\Delta^d_3\} = \{\Delta^s_3\} + \xi_3\{U_3\} \ ; \ \{\Delta^d_5\} = \{\Delta^s_5\} + \xi_5\{U_5\} \ ; \ \{\Delta^d_6\} = \{\Delta^s_6\} + \xi_6\{U_6\} \ ;$$
$$\{0\} = \{\Delta^s_8\} + \xi_7\{U_7\} \qquad (3.1)$$

Noting with $[K_{ij}]$ stiffness matrix of nodes $[K_{ij}] = [K_{ji}]$ and i, j that elastic equilibrium conditions of the nodes i, $i = 1,2,3,..,7$, are written as

$$[K_{11}]\{\Delta_1\} + [K_{12}]\left(\{\Delta_1\} - \{\Delta^s_2\}\right) + [K_{13}]\left(\{\Delta_1\} - \{\Delta^s_3\}\right) = \{0\} \ ;$$
$$[K_{12}]\left(\{\Delta^s_2\} - \{\Delta_1\}\right) + [K_{24}]\left(\{\Delta^d_2\} - \{\Delta_4\}\right) = \{0\}; [K_{13}]\left(\{\Delta^s_3\} - \{\Delta_1\}\right) + [K_{34}]\left(\{\Delta^d_3\} - \{\Delta_4\}\right) = \{0\} \qquad (3.2)$$
$$[K_{24}]\left(\{\Delta_4\} - \{\Delta^d_2\}\right) + [K_{34}]\left(\{\Delta_4\} - \{\Delta^d_3\}\right) + [K_{45}]\left(\{\Delta_4\} - \{\Delta^s_5\}\right) + [K_{46}]\left(\{\Delta_4\} - \{\Delta^s_6\}\right) = \{0\};$$

$$[K_{45}]\{\{\Delta_5^s\}-\{\Delta_4\}\}+[K_{57}]\{\{\Delta_5^d\}-\{\Delta_7\}\}=\{0\};[K_{46}]\{\{\Delta_6^s\}-\{\Delta_4\}\}+[K_{67}]\{\{\Delta_6^d\}-\{\Delta_7\}\}=\{0\}$$
$$[K_{57}]\{\{\Delta_7\}-\{\Delta_5^d\}\}+[K_{67}]\{\{\Delta_7\}-\{\Delta_6^d\}\}+[K_{78}]\{\{\Delta_7\}-\{\Delta_8^s\}\}=\{0\}.$$

and if one takes into account the relations (3.1) and the notation

$$\begin{gathered}[K_{11}]=[K_{1A}]+[K_{12}]+[K_{13}]\ ;\ [K_{22}]=[K_{21}]+[K_{24}]\ ;\ [K_{33}]=[K_{31}]+[K_{34}]\ ;\\ [K_{44}]=[K_{42}]+[K_{43}]+[K_{45}]+[K_{46}]\ ;\ [K_{55}]=[K_{54}]+[K_{57}]\ ;\ [K_{66}]=[K_{64}]+[K_{67}]\\ [K_{77}]=[K_{75}]+[K_{76}]+[K_{78}].\end{gathered} \tag{3.3}$$

results

$$\begin{gathered}[K_{11}]\{\Delta_1\}-[K_{12}]\{\Delta_2^s\}-[K_{13}]\{\Delta_3^s\}=\{0\}\ ;\\ [K_{21}]\{\Delta_1\}+[K_{22}]\{\Delta_2^s\}-[K_{24}]\{\Delta_1\}+\xi_2[K_{24}]\{U_2\}=\{0\}\ ;\\ [K_{31}]\{\Delta_1\}+[K_{33}]\{\Delta_3^s\}-[K_{34}]\{\Delta_4\}+\xi_3[K_{34}]\{U_3\}=\{0\}\\ [K_{42}]\{\Delta_2^s\}-[K_{43}]\{\Delta_3^s\}+[K_{44}]\{\Delta_4\}-[K_{45}]\{\Delta_5^d\}-[K_{46}]\{\Delta_6^s\}-\xi_2[K_{42}]\{U_2\}-\xi_3[K_{43}]\{U_3\}=\{0\}\\ [K_{45}]\{\Delta_4\}+[K_{55}]\{\Delta_5^s\}-[K_{57}]\{\Delta_7\}+\xi_5[K_{57}]\{U_5\}=\{0\}\ ;\\ [K_{46}]\{\Delta_4\}+[K_{66}]\{\Delta_6^s\}-[K_{67}]\{\Delta_7\}+\xi_6[K_{67}]\{U_6\}=\{0\}\ ;\\ [K_{57}]\{\Delta_5^s\}-[K_{67}]\{\Delta_6^s\}+[K_{77}]\{\Delta_7\}-\xi_5[K_{57}]\{U_5\}-\xi_6[K_{67}]\{U_6\}-\xi_8[K_{78}]\{U_8\}=\{0\}.\end{gathered} \tag{3.4}$$

If we note with $-\{E_2^s\}$ effort from the left section 2 and $\{R_2\}$ kinematic coupling reaction of the coupling when the condition of equilibrium results

$$-\{E_2^s\}+\{R_2\}=\{0\} \tag{3.5}$$

and given equal $[\tilde{U}_2]\{E_2^s\}=\{0\}$ or $[\tilde{U}_2]\{R_2\}=\{0\}$ we get the relation

$$[\tilde{U}_2][K_{12}]\{\Delta_1\}-[\tilde{U}_2][K_{13}]\{\Delta_2^s\}=\{0\} \tag{3.6}$$

Similarly longer obtain the equations

$$\begin{gathered}[\tilde{U}_3][K_{13}]\{\Delta_1\}-[\tilde{U}_3][K_{13}]\{\Delta_3^s\}=\{0\}\ ;[\tilde{U}_5][K_{45}]\{\Delta_4\}-[\tilde{U}_5][K_{45}]\{\Delta_5^s\}=\{0\}\ ;\\ [\tilde{U}_6][K_{46}]\{\Delta_4\}-[\tilde{U}_6][K_{46}]\{\Delta_6^s\}=\{0\}\ ;[\tilde{U}_7][K_{78}]\{\Delta_7\}+\xi_8[\tilde{U}_8][K_{78}]\{\tilde{U}_8\}=[\tilde{U}_8]\{F\}.\end{gathered} \tag{3.7}$$

The system consists of equations (3.4), (3.5), (3.7) is a system that contains 47 scalar equations with 47 unknown namely unknown 42 $\{\Delta_1\},\{\Delta_2^s\},\{\Delta_3^s\},\{\Delta_4\},\{\Delta_5^s\},\{\Delta_6^s\},\{\Delta_7\}$ and 5 unknown matrices $\xi_2,\xi_3,\xi_5,\xi_6,\xi_7$.

CONCLUSIONS

This approach makes it possible to develop mathematical model to calculate reaction forces in mechanisms gimballed and technological deviations, as will be shown in a following paper.

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